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MACROECONOMIC DYNAMICS MODELED IN SDEM-2: CAN SELF-INTERESTED BUSINESS PREFER STAGNATION TO GROWTH?

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1. Introductory notes

The Structural Dynamic Economic Model (SDEM) was initially proposed in (Barth, 2003) and later served as a basis for the development of the Multi-Actor Dynamic Integrated Assessment Model (MADIAM) aimed at modeling the dynamics of the coupled climate-socioeconomic system under conditions of global warming (Weber et al., 2005). In the current paper we present an upgraded model SDEM-2 and study the dynamics of a closed economy driven by conflict of interests of two model actors: employers (business) and employees (wage-earners).

2. SDEM-2: model description

2.1. Model variables. Dynamic equations

We consider the closed economy and assume exogenous growth of population $L(t)$ from the initial value L_0 at a constant rate λ_L:

$$L(t) = L_0 e^{\lambda_L t}. \quad (1)$$

Note that the population can exceed the employed labor since there might be unemployment in the economy. We then normalize all model variables by $L(t)$ and consequently have to increase the effective depreciation rates in the dynamic equations for per capita variables by λ_L (see below Eqs. (4)-(5)).

For per capita output y, we use the Leontief production function:

$$y = \min(\nu k, \mu h), \quad (2)$$

where k is per capita physical capital, h is per capita human capital, μ and ν are empirical factors. Eq. (2) implies that the level of human capital determines how much physical capital per employed person is needed to produce a unit of output. Thus, if $\nu k > \mu h$, there is idle physical capital in the model; while if $\nu k < \mu h$, there is unemployment. We use the term «balanced growth» for the dynamic regime when $\nu k = \mu h$ at every time moment.

We assume that output y is distributed between investment in physical capital y_k, investment in human capital y_h, and consumer good production g. The latter is further distributed between consumption of employees (wage-earners), equal to per capita wage rate w (normalized by the total population $L(t)$, not by the employed labor!) and the dividend d for employers (business):

$$y = y_k + y_h + g = y_k + y_h + w + d. \quad (3)$$

To be more precise, the dividend goes to the social group of employers, which is much smaller than the total working population; of interest is the dividend per employer. For the sake of simplicity, we assume that employers constitute a constant fraction of the (growing) population. In this case d in Eq. (3) is proportional to the dividend per employer.

The evolution of the economy is described by three dynamic equations:

$$\dot{k} = y_k - (\lambda_k + \lambda_L)k, \quad (4)$$

$$\dot{h} = y_h - (\lambda_h + \lambda_L)h + \zeta k, \quad (5)$$

$$\dot{w} = \lambda_w (w^{\text{targ}} - w). \quad (6)$$

Here λ_k is the depreciation rate of physical capital; λ_h is the human capital depreciation rate; ζk describes the «learning-by-doing» effect, where ζ is a constant empirical factor; λ_w is the wage adjustment rate, and w^{targ} is the (time-dependent) target wage rate.

Eq. (6) reflects the effects of wage negotiation process between employers and trade unions of employees, see discussion in (Barth, 2003; Weber et al., 2005). We assume that the target wage rate constitutes a constant fraction q of the maximum wage rate $w^{\max}(t)$ that is still possible to maintain the current state of the economy:

$$w^{\text{targ}}(t) = q w^{\max}(t), \quad 0 < q \le 1. \quad (7)$$

The maximum wage rate $w^{\max}$ in Eq. (7) in its turn, corresponds to an (imaginary) situation in which the investments y_k and y_h are at their minimum possible level $y_{k,\min}$, $y_{h,\min}$ that still maintains a non-decaying (i.e. invariable) level of physical and human capital ($\dot{k} = 0$, $\dot{h} = 0$), and there is no dividend for business:

$$w^{\max} = y - y_{k,\min} - y_{h,\min}. \quad (8)$$

To close the dynamics of the economy, business has to choose the (time-dependent) values of the control variables of the model, i.e. the investments in physical and human capital ($y_k(t)$ and $y_h(t)$, respectively).

2.2. Balanced growth path

In the present paper we consider only the particular case of balanced growth (see the definition in Sec. 2.1) in which Eq. (2) takes the form

$$y = \nu k = \mu h. (9)$$

It is assumed that business pursues certain «socially-oriented» goals and chooses the investments y_k and y_h at every time moment such that the balance of Eq. (9) is maintained, thereby avoiding both idle physical capital and unemployment. This means, of course, that investments y_k and y_h, and therefore also the two forms of capital (k and h), are no longer independent. Consequently, Eq. (5) for human capital dynamics can be eliminated from the system of model equations, and only one independent control variables remains in the model setup. We choose the dividend d as this variable. Note that Eq. (9) implies that both k and h are proportional to per capita GDP.

Under the assumptions of balanced growth, we arrive after some straightforward algebra to a simple parameterization for $w^{\max}$ appearing in Eq. (7):

$$w^{\max} = \varpi k\,, \qquad (10)$$

where a new constant is introduced for brevity:

$$\varpi \equiv \varsigma + \nu - \frac{\nu}{\mu}\lambda_h - \lambda_k - \left(1+\frac{\nu}{\mu}\right)\lambda_L\,. \qquad (11)$$

To ensure that in Eq. (5) $y_h > 0$, and in Eq. (10) $w^{\max} > 0$, we impose the restriction $\varpi > 0$ and also

$$\gamma \equiv \frac{\nu}{\mu}(\lambda_k + \lambda_L) - \varsigma > 0\,. \qquad (12)$$

Finally, after introducing for brevity the constant

$$\xi \equiv \frac{1}{1+\dfrac{\nu}{\mu}} \qquad (13)$$

(note that $\xi < 1$), the original three-dimensional dynamical system (Eqs. (4)-(6)) is reduced to the two-dimensional system

$$\dot{k} = \xi(\varpi k - w - d)\,, \qquad (14)$$

$$\dot{w} = \lambda_w (q\varpi k - w)\,. \qquad (15)$$

Let k_0, w_0 be the initial conditions of the problem. We impose the restrictions:

$$k_0 > 0\,; \quad w_0 > 0\,; \qquad (16)$$

$$0 \le d(t) \le d_{\max}(t) = \varpi k(t) - w(t)\,. \qquad (17)$$

The second inequality in Eq. (17) ensures that the economy does not decay ($\dot{k} \ge 0$); which, in combination with the first inequality from Eq. (16), implies that $k(t) > 0$ for all t. The validity of another restriction, $w(t) \ge 0$, follows directly from Eq. (15).

To fulfill Eq. (17) initially, the initial conditions must satisfy the inequality

$$\varpi k_0 - w_0 \ge 0\,. \qquad (18)$$

Finally, we rewrite the dynamic Eqs. (14) – (15) in matrix form:

$$\dot{\mathbf{x}} = \hat{\mathbf{A}}\mathbf{x} - \mathbf{e}_1 \xi d\,, \qquad \mathbf{x}(0) = \mathbf{x}_0\,, \qquad (19)$$

where:

$$\mathbf{x} = \begin{pmatrix} k \\ w \end{pmatrix}, \quad \mathbf{x}_0 = \begin{pmatrix} k_0 \\ w_0 \end{pmatrix},$$

$$\hat{\mathbf{A}} = \begin{pmatrix} \xi\varpi, & -\xi \\ q\lambda_w\varpi, & -\lambda_w \end{pmatrix}, \quad \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \qquad (20)$$

To close the system, we have to specify the business choice of dividend $d(t)$ under the restrictions imposed by Eqs. (16)-(17). In the following section, the development of the economy is modeled in a system-dynamics setup, while the same problem is treated in Sec. 4 in an optimization framework.

3. System dynamic setup. Business control strategies

In this section we study the economy described by the system of equations (14)–(15) (or by Eq. (19)) in a system dynamics mode. Following an approach proposed in (Weber et al., 2005) we assume that at every time moment business chooses the value of its dividend as a model control variable in accordance with a control strategy determined by the current state of the economy:

$$d(t) = d(\mathbf{x}(t))\,. \qquad (21)$$

This leads to a description of the evolution of the economy in system dynamics spirit. We emphasize that the approach does not imply any formal utility maximization procedure. The control strategy of business represents an intuitive decision process guided by some generally perceived goals, without support from a quantitative inter-temporal analysis. This corresponds to the model of a complex business environment in which the economic players have only limited information and foresight. The alternative inter-temporal optimization approach is considered below in Sec. 4.

We consider three different business control strategies: the «altruistic» strategy, the «moderate growth» strategy and the «here and now» strategy.

3.1. «Altruistic» strategy

We consider first an (unrealistic) case in which altruistic business strives only to maximize the growth of the economy by maximizing investment, taking no dividend:

$$d(t) = 0\,. \qquad (22)$$

The dynamic system (Eq. (19)) then takes the homogenous form

$$\dot{\mathbf{x}} = \hat{\mathbf{A}}\mathbf{x} \qquad (23)$$

with the solution

$$\mathbf{x}(t) = e^{\hat{\mathbf{A}}t}\mathbf{x}_0\,. \qquad (24)$$

The dynamic properties of the solution depend on the signs of the eigenvalues $\lambda_\pm$ of the matrix $\hat{\mathbf{A}}$, which satisfy the conditions

$$\lambda_+ + \lambda_- = \mathrm{Sp}\,\hat{\mathbf{A}} = \xi\varpi - \lambda_w\,, \qquad (25)$$

$$\lambda_+\lambda_- = \det\hat{\mathbf{A}} = (q-1)\lambda_w\xi\varpi\,. \qquad (26)$$

The r.h.s. of Eq. (25) can be positive, zero or negative, depending on the values of model parameters. The r.h.s. of Eq. (26) is equal to zero when $q = 1$ and negative when $q < 1$. This means that if $q < 1$ the eigenvalues λ_+ and λ_- always have opposite signs: we assume for this case that $\lambda_+ > 0 > \lambda_-$. In case of $q = 1$ one eigenvalue is equal to zero, whereas the other is equal to the r.h.s. of Eq. (25) and therefore can be positive, zero or negative, depending on the values of model parameters.

Explicitly,

$$\mathbf{x}(t) = \begin{pmatrix} k(t) \\ w(t) \end{pmatrix} = C_+\mathbf{s}_+ e^{\lambda_+ t} + C_-\mathbf{s}_- e^{\lambda_- t}\,, \qquad (27)$$

where $\mathbf{s}_\pm$ are the corresponding eigenvectors and $C_\pm$ are scalar constants depending on the initial conditions (for completeness sake, we mention that Eq. (27) is not applicable for the particular case $\lambda_+ = \lambda_- = 0$, for which linear growth takes place).

In case of $q < 1$ we have $\lambda_+ > 0 > \lambda_-$, and Eq. (27) describes the exponential growth $\sim e^{\lambda_+ t}$ in the asymptotic limit.

In case of $q = 1$ three different dynamic regimes are formally possible: (i) exponential growth ($\lambda_+ = \xi\varpi - \lambda_w > 0$, $\lambda_- = 0$); (ii) linear growth ($\xi\varpi = \lambda_w$, $\lambda_+ = \lambda_- = 0$); (iii) growth leading to stagnation ($\lambda_+ = 0$, $\lambda_- = \xi\varpi - \lambda_w < 0$). In the latter case, asymptotically there is no growth, and $k(t)$, $w(t)$ converge to finite constant limits. We note however that the case $q = 1$ describes an artificial situation of extreme negotiation power of trade unions (see Eq. (7): $w^{targ}(t) = w^{max}(t)$ if $q = 1$). Therefore in what follows we make calculations and provide simulation results for the case $q < 1$.

The dynamics of $k(t)$ and $w(t)$ under the «altruistic» strategy adopted is illustrated by Fig. 1 for two different values of q.

3.2. «Moderate growth» strategy

We now assume that $q < 1$ and business chooses its dividend in such a way that the steady growth of per capita physical capital (and consequently GDP) at a rate β ($0 < \beta < \lambda_q$) is maintained. The dynamic system of Eqs. (14) – (15) yields in this case:

$$k = k_0 e^{\beta t}, \qquad (28)$$

$$w(t) = \frac{\lambda_w}{\lambda_w + \beta} q\varpi k_0 e^{\beta t} + \left[w_0 - \frac{\lambda_w}{\lambda_w + \beta} q\varpi k_0 \right] e^{-\lambda_w t}, \quad (29)$$

$$d(t) = -\frac{\Delta(\beta)}{\xi(\lambda_w + \beta)} k_0 e^{\beta t} - \left[w_0 - \frac{\lambda_w}{\lambda_w + \beta} q\varpi k_0 \right] e^{-\lambda_w t}, \quad (30)$$

where $\Delta(\beta) \equiv \det(\hat{\mathbf{A}} - \beta\hat{\mathbf{I}})$, $\hat{\mathbf{I}}$ being the unit matrix. It can be easily checked that in case $0 < \beta < \lambda_q$ we have $\Delta(\beta) < 0$, and $d(t) \to +\infty$ when $t \to +\infty$. Note also that in the case under consideration the initial conditions should satisfy a more rigid inequality than Eq. (18), namely

$$\varpi k_0 - w_0 \geq \frac{\beta}{\xi} k_0 \iff \beta \leq \xi \left(\varpi - \frac{w_0}{k_0} \right). \qquad (31)$$

To sum up, in case of the «moderate growth» strategy the per capita physical capital grows exponentially (although less rapidly than when the «altruistic» strategy is adopted). The wage rate and the dividend also grow exponentially in the asymptotic limit at the rate β (Fig. 2).

3.3. «Here and now» strategy

Finally, we consider the «moderate growth» control strategy in the limiting case when $\beta = 0$, i. e. when the per capita physical capital does not grow at all:

$$k(t) = \text{const} = k_0. \qquad (32)$$

This situation corresponds to the case in which business decides to choose at every time instant the maximum dividend possible for the current state of the economy («here and now» strategy):

$$\dot{k} = 0, \; d(t) = d_{\max}(t) = \varpi k(t) - w(t). \qquad (33)$$

Eqs. (29)-(30) in this case take the form

$$w(t) = q\varpi k_0 + (w_0 - q\varpi k_0) e^{-\lambda_w t}, \qquad (34)$$

$$d(t) = (1 - q)\varpi k_0 - (w_0 - q\varpi k_0) e^{-\lambda_w t}. \qquad (35)$$

As evident from Eqs. (32), (34)-(35), the «here and now» strategy leads to stagnation: neither the per capita physical capital nor GDP grows, while the wage rate and dividend converge asymptotically to constant values. Moreover, in the particular case $q = 1$ the business dividend converges to zero in the long term.

However, as we will see in the next section, in a rather natural optimization model framework with a linear utility function, and for the case of a sufficiently large discount rate, self-interested business striving to maximize its utility will indeed choose this «here and now» strategy, leading to the socially non-optimal situation of stagnation.

4. Optimization approach

We now switch to the optimization model framework and assume that business chooses the time path for dividend that maximizes its utility defined as the discounted dividend:

$$\max U = \int_0^T d(t) e^{-\delta t} dt, \qquad (36)$$

where T is the terminal time and δ the discount rate. The time path for the dividend has to fulfill the restrictions of Eqs. (16) – (17).

The main finding is that for sufficiently large discount rates (more precisely, for $\delta > \lambda_q$) business will prefer the «here and now» strategy, choosing at every time moment $d(t) = d_{\max}(t) = \varpi k(t) - w(t)$ (see Eq. (33)). This strategy leads to stagnation and, as mentioned above, is, of course, not favored by wage-earners. Heuristically, the rationale for this strategy can be understood from Fig. 2*c* (see the caption to the figure): in order to maintain even moderate growth of the economy, business would have to sacrifice a substantial part of its dividend at the initial phase of development.

We omit the proof of the optimality of «here and now» strategy for business at sufficiently large discount rates because of space constraints. The proof is essentially based on the linear dependence of utility function on model control variable (dividend d) – see Eq. (36). In a follow-up paper, we are planning to consider the case of logarithmic utility function

$$\tilde{U} = \int_0^T \ln(d(t)) e^{-\delta t} dt \qquad (37)$$

and check the robustness of solution of optimization problem to the structure of the objective function.

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FIGURES

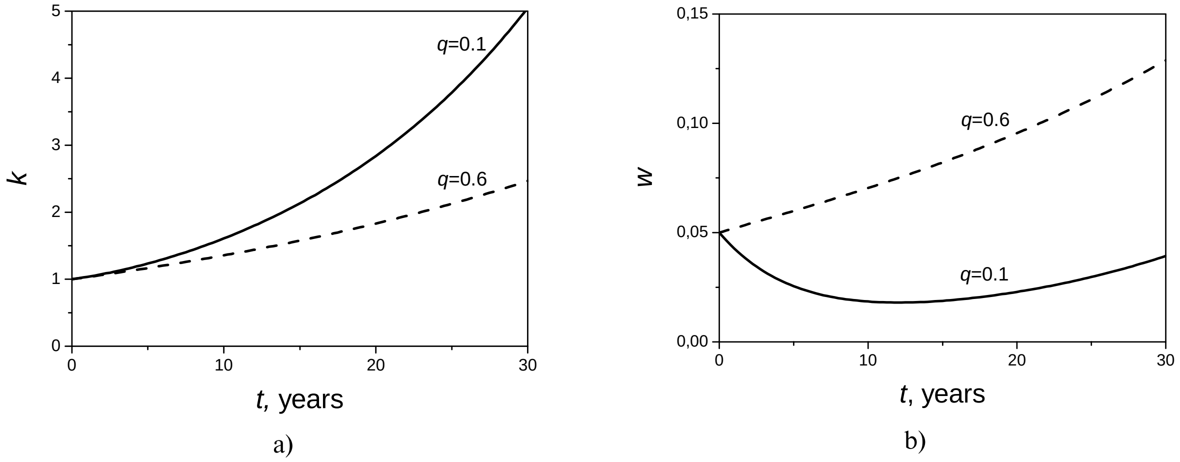

Fig. 1. Per capita physical capital *k* (proportional to per capita GDP, *a*) and wage rate *w* (*b*) in case of «altruistic» business control strategy ($d(t) \equiv 0$) for two different values of parameter *q* characterizing the wage negotiation power of trade unions (larger values of *q* correspond to more negotiation power). The exponential growth takes place asymptotically.

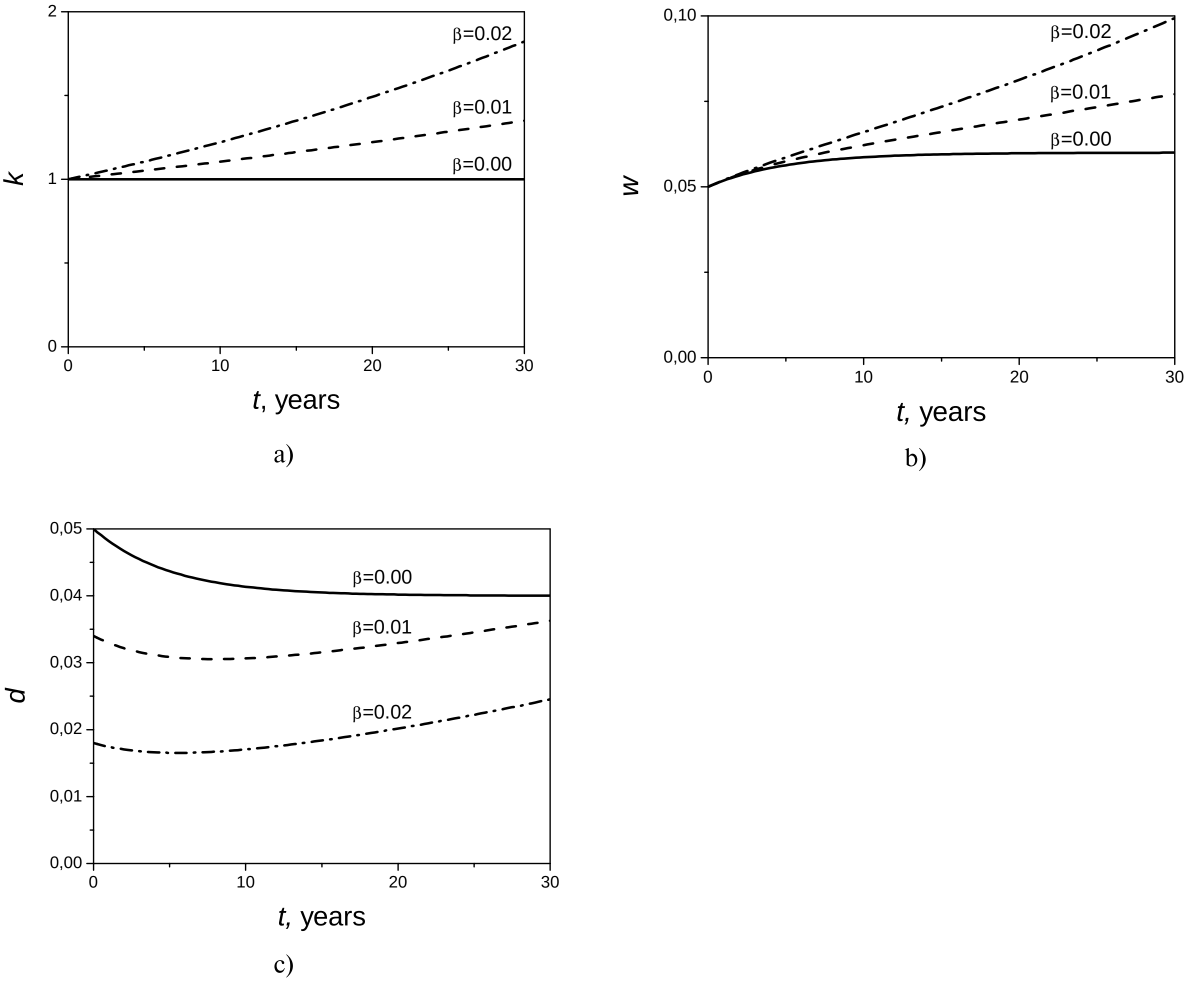

Fig. 2. Per capita physical capital *k* (*a*), wage rate *w* (*b*), and dividend *d* (*c*) for «here and now» ($\beta = 0.00$) and «moderate growth» ($\beta = 0.01$ $\beta = 0.02$,) business control strategies. Wage-earners would definitely prefer the «moderate growth» strategy; however to maintain the latter business would have to sacrifice a substantial fraction of its dividend at the initial stage of development. In the optimization model setup and for sufficiently large discount rates ($\delta > \lambda_+$) business will choose the «here and now» strategy leading to stagnation of the economy (see Sec. 4).